Method for Evaluating Gravity Effects in the Testing of Nutation Dampers

Kyle T. Alfriend*
Naval Research Laboratory, Washington, D.C.

A method is developed for determining the effect of gravity in the testing of nutation dampers on symmetric single or dual spin satellites. The basic theory is developed and then applied to the partially filled viscous ring damper and the spring-mass-dashpot damper. A comparison with test results for the viscous ring damper is also given.

Introduction

IN assessing the performance of a nutation damper on a spinning satellite from dynamic tests the effect of gravity on the test data is usually ignored. This would seem to be a reasonable assumption since the spacecraft is statically balanced prior to any test, and the effect of any small imbalance should cause only a small oscillation in the nutation angle as occurs in the motion of a spinning top. However, if the gravitational force alters the motion of the damper the rate of energy dissipation is changed which results in a change in the time constant. In this paper a method is developed for determining the effect of gravity on the nutational motion of a symmetric single or dual spin satellite with a nutation damper. The only restriction is that the nutation damper can be modeled as a rigid body. The result is expressed as a constant which when multiplied by the time constant for $g=0$ gives the time constant for $g\neq 0$. An analysis is given for the effect of gravity on two nutation dampers, the partially filled viscous ring damper and the spring-mass-dashpot damper perpendicular to the spin axis. A comparison with computer simulations and test results for the partially filled viscous ring damper is also given.

This study resulted from an analysis of the test data from tests run at NASA/GSFC on the Helios satellite which was a symmetric spinning satellite with a partially filled viscous ring damper. After looking at the test data it appeared that gravity was having a substantial effect on the motion of the satellite because as the spin rate was increased the time constant increased rather than decreased as expected. The subsequent analysis [1] showed that in several cases the effect of gravity was to change the time constant by a factor greater than 100.

Basic Theory

Consider a symmetric dual spin satellite with total angular momentum $\mathbf{H}=H_x\mathbf{e}_x+H_y\mathbf{e}_y+H_z\mathbf{e}_z$ where x, y, and z are principal axes and z is the spin axis. H_x, H_y, and H_z are given by

$$H_x=A(p+\epsilon h_x) \tag{1a}$$

$$H_y=A(q+\epsilon h_y) \tag{1b}$$

$$H_z=A(\sigma r+\bar{h}+\epsilon h_z) \tag{1c}$$

where A is the transverse inertia, σ is the inertia ratio, $A\bar{h}$ is the momentum of the rotor, ϵ is a suitable small parameter such as the ratio of the damper mass to the satellite mass, and $A\epsilon h_x$, $A\epsilon h_y$, $A\epsilon h_z$ are the contributions of the damper to the angular momentum. It is assumed that the satellite is statically balanced so that any external torque is a result of the gravitational force acting on the damper. This torque $\bar{M}$ on the system is

$$\bar{M}=\epsilon AM \tag{2}$$

The nutation angle θ, which is the angle between the spin axis and H is given by

$$\tan\theta=[(H_x^2+H_y^2)^{1/2}/H_z] \tag{3}$$

Differentiating Eq. (3), substituting Eq. (1), and expanding in a power series in ϵ yields

$$\dot{\theta}=\frac{\epsilon(ph_y-qh_x)}{(\sigma r+\bar{h})\tan\theta}+\frac{\epsilon(pM_x+qM_y-(\sigma r+\bar{h})M_z\tan^2\theta)}{(\sigma r+\bar{h})^2\tan\theta\sec^2\theta}+O(\epsilon^2) \tag{4}$$

Since $\dot{\theta}=O(\epsilon)$ we only need p, q, and r to $O(\epsilon^\circ)$ to obtain θ to $O(\epsilon)$. Hence we can obtain p, q, and r from the equations of motion for $\epsilon=0$, i.e., the equations of motion for a torque free dual spin satellite, which are

$$\dot{p}+(\lambda r+\bar{h})q=0 \tag{5a}$$

$$\dot{q}-(\lambda r+\bar{h})p=0 \tag{5b}$$

$$\sigma\dot{r}+\dot{\bar{h}}=0 \tag{5c}$$

Assuming the rotor momentum to be constant gives

$$p=-\omega_t\cos\lambda_1 t \tag{6a}$$

$$q=-\omega_t\sin\lambda_1 t \tag{6b}$$

$$\sigma r+\bar{h}=\text{const.} \tag{6c}$$

where

$$\lambda_1=\lambda r+\bar{h} \tag{7a}$$

$$\lambda=\sigma-1 \tag{7b}$$

$$\omega_t=(\sigma r+\bar{h})\tan\theta+O(\epsilon) \tag{7c}$$

Substituting Eq. (6) into Eq. (4) yields

$$\dot{\theta}=\epsilon(h_x\sin\lambda_1 t-h_y\cos\lambda_1 t)-(\epsilon\cos^2\theta/(\sigma r+\bar{h})]\times(M_x\cos\lambda_1 t+M_y\sin\lambda_1 t+M_z\tan\theta)+O(\epsilon^2) \tag{8}$$

Received September 6, 1974; revision received January 1, 1975. This research was supported by the NASA Goddard Space Flight Center under NASA Grant NGR-33-010-169.
Index category: Spacecraft Attitude Dynamics and Control.
*Space Systems Division. Member AIAA.

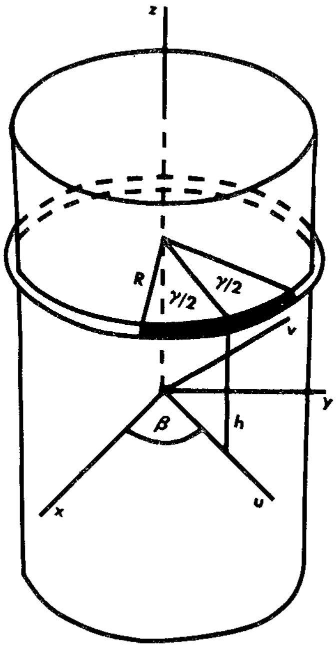

Fig. 1 Mathematical model for the partially filled viscous ring damper.

The effect of gravity on θ for a particular damper is obtained by determining h_x, h_y, h_z, M_x, M_y, and M_z for the damper and solving Eq. (8). Since we are just obtaining θ to $O(\epsilon)$ only the damper motion for $\epsilon=0$ is needed and this is relatively simple to obtain. As will be seen later, an assumption needed in solving for θ is that the change in direction of $\boldsymbol{H}$ is $O(\epsilon)$. The validity of this assumption is checked by computer simulation for the partially filled viscous ring damper. Although Eq. (8) is valid for large θ in most tests θ is small; hence Eq. (8) can usually be linearized in θ.

Applications

Partially Filled Viscous Ring Damper

Consider an axisymmetric satellite (see Fig. 1) spinning about its axis of symmetry at a rate Ω. Encircling the spin axis at a height h above the center of mass is a tube of radius R which is partially filled with a fluid, the angle of fill being γ. Modeling the motion of the fluid as a rigid slug, it has been shown[1-3] that there are two distinct modes of motion called the nutation synchronous mode and the spin synchronous mode. In the nutation synchronous mode, which occurs for "large" nutation angles, the centrifugal force is the dominant force acting on the slug. This causes the center of mass of the slug to be almost aligned with the transverse angular velocity vector ω_t, hence the slug rotates at the constant rate of $(1-\sigma)\Omega$ with respect to the satellite. Since a component of the centrifugal force is needed to balance the friction force the center of mass of the slug is slightly off ω_t. It was shown by Alfriend[1,2] that in this mode the nutation angle time history is given by

$$\cos\theta=\cos\theta_0\exp(t/t_{cn}) \tag{9}$$

$$t_{cn}=\sigma/[\epsilon\eta\bar{\gamma}(\sigma-1)\Omega] \tag{10}$$

where $\bar{\gamma}$ is the fraction fill, $\bar{\gamma}=\gamma/2\pi$, η is a dimensionless damping parameter, m is the mass of the slug and ϵ is the ratio of the enertia of a tube filled with fluid to the transverse moment of inertia of the satellite, i.e.,

$$\epsilon=(mR^2/A\bar{\gamma}) \tag{11}$$

As the nutation angle decreases, the slug must move further from ω_t to balance the friction force. Finally there becomes a point where it can no longer balance the friction force and the slug begins to be dragged around with the tube while oscillating in the tube. This mode of motion is called the "spin synchronous mode" and the nutation angle time history is given by

$$\theta=\theta_0\exp(-t/t_{cs}) \tag{12}$$

$$t_{cs}=2(\sigma-1)[\sigma-1)^2+\eta^2]/(\epsilon b^2\sigma^3\eta\bar{\gamma}k^2\Omega) \tag{13}$$

$$k=[\sin\gamma/2)]/(\gamma/2) \tag{14}$$

The transition angle θ_t between the two modes is given by

$$\tan\theta_t=(\eta|\sigma-1|/bk\sigma^2) \tag{15}$$

From Ref. 1 we have

$$h_y/\bar{\gamma}=(b^2+\tfrac{1}{2})p+[-p\sin2\beta+q\cos2\beta](\sin\gamma/\gamma)/2 - bk(r+\dot{\beta})\cos\beta \tag{16a}$$

$$h_y/\bar{\gamma}=(b^2+\tfrac{1}{2})q + [-p\sin2\beta+q\cos2\beta](\sin\gamma/\gamma)/2 - bk(r+\dot{\beta})\sin\beta \tag{16b}$$

$$h_z/\bar{\gamma}=r+\dot{\beta}-bk(p\cos\beta+q\sin\beta) \tag{16c}$$

Letting ψ be the spin angle the moment $\boldsymbol{M}$ due to the gravitational force is

$$M_x=\bar{g}\Omega^2\bar{\gamma}(b\sin\theta_g\cos\psi-k\cos\theta_g\sin\beta) \tag{17a}$$

$$M_y=\bar{g}\Omega^2\bar{\gamma}(-b\sin\theta_g\sin\psi+k\cos\theta_g\cos\beta) \tag{17b}$$

$$M_z=-\bar{g}\Omega^2\bar{\gamma}k\sin\theta_g\cos(\psi+\beta) \tag{17c}$$

where Ω is the spin rate and $\bar{g}$ is the ratio of the gravitational acceleration to the centrifugal acceleration, i.e.,

$$\bar{g}=g/R\Omega^2 \tag{18}$$

and θ_g is the angle between the spin axis and local vertical. Substituting Eqs. (16) and (17) into Eq. (8) gives

$$\dot{\theta}=\epsilon\bar{\gamma}[bk(\Omega+\dot{\beta})-\omega_t(\sin\gamma/\gamma)\cos(\beta-\lambda\Omega t)]\sin(\beta-\gamma\Omega t) + (\epsilon\bar{\gamma}k\bar{g}\Omega/\sigma)\sin(\beta-\lambda\Omega t)\cos\theta\cos(\theta-\theta_g)$$

The missing ingredient is the solution for β. From Ref. 1 the differential equation for β with $\epsilon=0$ is

$$\ddot{\beta}+\eta\Omega\dot{\beta}-(\sin\gamma/2\gamma)\omega_t^2\sin2(\beta-\lambda\Omega t) + b\sigma k\Omega\omega_t\sin(\beta-\lambda\Omega t)+k\Omega^2\bar{g}\sin\theta_g\cos(\beta+\psi)=0 \tag{20}$$

Nutation Synchronous Mode

Let α be the angle between ω_t and the vector from the spin axis to the center of mass of the slug, i.e.,

$$\alpha=\beta-\lambda\Omega t \tag{21}$$

Assuming that ψ can be represented by its solution for torque free motion, that is, $\psi=-\lambda\Omega t-\pi/2$, and using $\omega_t=\sigma\Omega\tan\theta$ the differential equation for α becomes

$$\ddot{\alpha}+\eta\Omega\dot{\alpha}+b\sigma^2\Omega^2k\tan\theta\sin\alpha\left[1+\frac{\bar{g}}{b\sigma^2}\frac{\sin\theta_g}{\tan\theta} - \cos(\gamma/2)\tan\theta\cos\alpha\right]=-\eta\lambda\Omega^2 \tag{22}$$

Using the assumption that $(\theta-\theta_g)=O(\epsilon)$ there is an equilibrium value $\alpha=\alpha_e$ given by

$$\sin\alpha_e\left[1+\frac{\bar{g}}{b\sigma^2}\cos\theta-\frac{\cos(\gamma/2)}{b}\tan\theta\cos\alpha_e\right]=-\eta\lambda/b\sigma^2\tan\theta \tag{23}$$

Substituting Eqs. (21) and (23) into Eq. (10) gives

$$\tan\theta\dot{\theta}=-\epsilon(\eta\bar{\gamma}(\sigma-1)\Omega/\sigma) \tag{24}$$

Since $\bar{g}$ does not appear in Eq. (24) we conclude that gravity has no effect on the motion in the nutation synchronous mode.

The comparison of t_{cn} and an exact† time constant obtained by numerical integration of the equations of motion shown in Fig. 2a shows that the approximate solution is a good approximation. The numerical integration also showed that the assumption $(\theta-\theta_g)=O(\epsilon)$ is a valid assumption.

The transition angle between the two modes is obtained by setting $\alpha_e=\pm\pi/2$ which for small angles becomes

$$\theta_T=(\eta|\sigma-1|/b\sigma^2kG) \tag{25}$$

where

$$g=1+\bar{g}/\sigma^2b \tag{26}$$

Thus the only effect gravity has on the nutation synchronous mode is to decrease the transition angle.

Spin Synchronous Mode

Using the assumptions that ψ can be represented by the solution for torque free motion and that $(\theta-\theta_g)=O(\epsilon)$ the equation of motion for β is

$$\ddot{\beta}+\eta\Omega\dot{\beta}+b\sigma^2\Omega^2k\tan\theta\left[G-\frac{\cos(\gamma/2)}{b}\times\cos(\beta-\lambda\Omega t)\tan\theta\right]\sin(\beta-\lambda\Omega t)=0 \tag{27}$$

Assuming that θ is small and using the first iterate of a Picard iteration scheme (the procedure used in Refs. 1, 2) a good first approximation of the solution of Eq. (27) is

$$\beta-\beta_0=\left(\frac{bk\sigma^2G\theta}{\lambda(\lambda^2+\eta^2)}\right)[\lambda\sin(\beta_0-\lambda\Omega t)-\eta\cos(\beta_0-\lambda\Omega t)] \tag{28}$$

where β_0 is the initial value of β. Substitution of Eq. (28) into Eq. (19) and using $\sin(\beta-\beta_o)=\beta-\beta_0$ and $\cos(\beta-\beta_o)=1$ gives

$$\dot{\theta}+\left[\frac{1}{t_{cs}}+\text{oscillatory terms}\right]\theta=-\epsilon\bar{\gamma}bk\Omega G\sin\lambda\Omega t \tag{29}$$

where

$$t_{cs}=\frac{2(\sigma-1)[(\sigma-1)^2+\eta^2]}{\epsilon b^2\sigma^3\eta\bar{\gamma}k^2\Omega G^2}=\frac{t_{cs}(\text{no gravity})}{G^2} \tag{30}$$

Thus gravity can have a substantial effect on the nutation angle in the spin synchronous mode as the time constant is reduced by a factor $1/G^2$.

A comparison of t_{cs} given by Eq. (30) and the time constant obtained from numerical integration is given in Fig. 2b. It is

†The exact time constant is obtained by integrating the exact equations over a suitable period of time assuming the approximate solution and calculating the time constant.

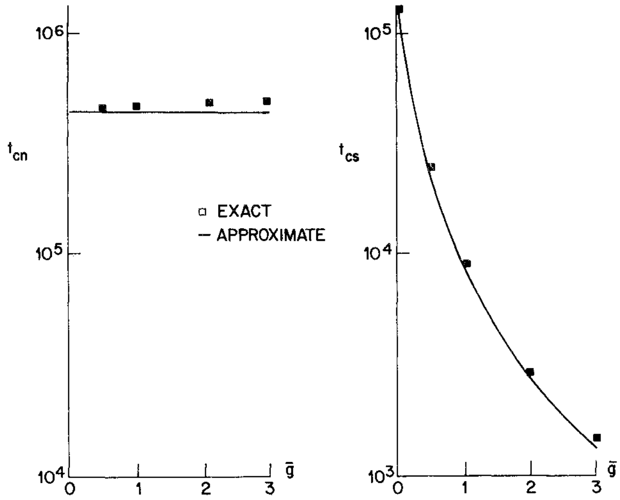

Fig. 2 Comparison of exact and approximate time constants.

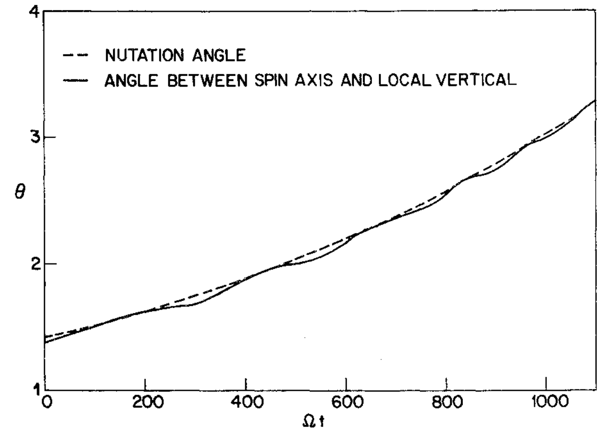

Fig. 3 Check on the assumption $(\theta-\theta_g)=O[\epsilon]$.

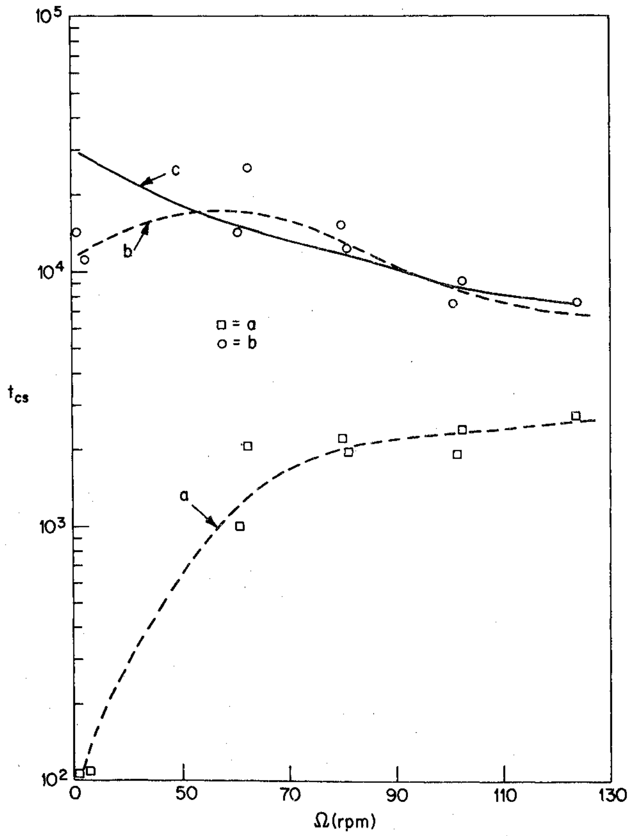

Fig. 4 Comparison between predicted and test results.

seen that the agreement is good. Figure 3 shows the validity of the assumption $(\theta - \theta_g) = O(\epsilon)$

Comparison with Test Results

In October 1972 a series of tests were run at NASA/GSFC on the Helios damper. A total of 36 tests were run with four inertia ratios and two damper locations. In all the tests the damper was offset 0.25 in from the spin axis to insure that the fluid behaved as a rigid slug. The tests results are given in Hraster,[4] and an analysis of the test results in Alfriend.[1] In all the tests the nutation angle time history appeared to have an exponential behavior, hence it is assumed the satellite was in the spin synchronous mode. Thus there are no results for the nutation synchronous mode. Figure 4 which is a plot of t_{cs} vs Ω for one series of tests gives a comparison between predicted results and test results. Curve *a* is a plot of the test results, curve *b* is curve *a* with the effect of gravity removed using Eq. (30) and it shows the tremendous effect gravity has on the test results. Curve *c* is a plot of Eq. (13) using a constant value of η obtained from the tests. (The predicted value of η was larger by a factor of 4-5, see Ref. 1). Comparison of curves (*b*) and (*c*) shows that the method developed has effectively removed the effect of gravity from the data.

Spring-Mass-Dashpot Nutation Damper

Consider a single spin symmetric satellite with a spring-mass-dashpot nutation damper mounted perpendicular to the spin axis at a height b above the center of mass in the x-y plane. The spring constant is k, the damping constant of the dashpot of c and the spring has no force when the mass is on the spin axis. Letting the deflection of the spring be $b\xi$ the position vector $\boldsymbol{r}$ and the velocity $\boldsymbol{v}$ of the mass are

$$\boldsymbol{r} = b[\xi \boldsymbol{e}_x + \boldsymbol{e}_z] \tag{31}$$

$$v = b[(\dot{\xi} + q = \boldsymbol{e}_x + r\xi - p)\boldsymbol{e}_y - q\xi \boldsymbol{e}_y] \tag{32}$$

The normalized angular momentum and torque are

$$h_x = p - \xi r \tag{33a}$$

$$h_y = \dot{\xi} + q(1 + \xi^2) \tag{33b}$$

$$h_z = \xi^2 r - p\xi \tag{33c}$$

$$M_x = \bar{g}\Omega^2 \sin\theta_g \cos\psi \tag{34a}$$

$$M_y = \bar{g}\Omega^2 (\xi\cos\theta_g - \sin\theta_g \sin\psi) \tag{34b}$$

$$M_z = -\bar{g}\Omega^2 \xi \sin\theta_g \cos\psi \tag{34c}$$

where

$$\bar{g} = g / b\Omega^2 \tag{35a}$$

$$\epsilon = mb^2 / A \tag{35b}$$

and Ω is the spin rate.

Substituting Eqs. (33) and (34) into Eq. (8), using the assumption that $\psi = -\lambda\Omega t - \pi/2$ and $(\theta - \theta_g) = O(\epsilon)$, and neglecting nonlinear terms in ξ since ξ is proportional to θ gives

$$\dot{\theta} = -\epsilon[(1 + \bar{g}/\sigma)\Omega\xi \sin\lambda\Omega t + \dot{\xi}\cos\lambda\Omega t] \tag{36}$$

Using Lagrange's equations the equation of motion for the mass for small nutation angles and small deflections ($\xi << 1$) is

$$\ddot{\xi} + \bar{c}\dot{\xi} + K\xi = \sigma^2\Omega^2 G\theta \cos\lambda\Omega t \tag{37}$$

which has the steady-state solution

$$\xi = SG\theta[(K - \lambda^2\Omega^2)\cos\lambda\Omega t + \bar{c}\lambda\Omega \sin\lambda\Omega t] \tag{38}$$

where

$$K = (k/mb^2) - \Omega^2 \tag{39a}$$

$$\bar{c} = (c/mb) \tag{39b}$$

$$G = 1 + \bar{g}/\sigma^2 \tag{39c}$$

$$S = \sigma^2\Omega^2 / [(K - \lambda^2\Omega^2)^2 + (\bar{c}\lambda\Omega)^2] \tag{39d}$$

Substituting Eq. (37) into Eq. (36) gives

$$\dot{\theta} + (1/t_c + \text{oscillatory terms})\,\theta = 0 \tag{40}$$

where

$$t_c = \left\{ \frac{2[(K - \lambda^2\Omega^2)^2 + (\bar{c}\lambda\Omega)^2]}{\epsilon\sigma^3\Omega^4\lambda\bar{c}} \right\} \frac{1}{G^2} \tag{41}$$

Thus the time constant for a test would differ from the flight time constant by a factor $(1/G^2)$.

Summary

A method which evaluates the effect of gravity on the testing of nutation dampers for spinning satellites has been developed. When applied to a specific problem the result is a constant which when multiplied by the time constant for flight gives the time constant for the test. This multiplicative constant is a function of gravity and the system parameters. The method is useful in the analysis of test results and also in the planning of tests.

The method has been applied to two dampers, the partially filled viscous ring damper and the spring-mass-dashpot damper mounted perpendicular to the spin axis. A comparison with tests results for the viscous ring damper has also been given. For the viscous ring damper it was shown that gravity does not affect the time constant in the nutation synchronous mode but that it can affect appreciably the time constant in the spin synchronous mode. The change in the time constant is

$$t_{cs} = t_{cs}(\text{no gravity})/(1 + \bar{g}/\sigma^2 b)^2 \tag{42}$$

The effect of gravity on the time constant for the spring-mass-dashpot damper is similar to the effect in the spin synchronous mode of the viscous ring damper.

$$t_c = t_c(\text{no gravity})/(1 + \bar{g}/\sigma^2)^2 \tag{43}$$

References

[1] Alfriend, K.T., *Analysis of the Partially Filled Viscous Ring Damper,* Final Report NASA Grant NGR-33-010-169, CR-138488, Oct. 1973, NASA.

[2] Alfriend, K.T., "The Partially Filled Viscous Ring Damper," *Journal of Spacecraft and Rockets,* Vol. 11, July 1974, pp. 456-462.

[3] Cartwright, W.F., Massingill, E.C., and Trueblood, R.D., "Circular Constraint Nutation Damper," *AIAA Journal,* Vol. 1, June 1963, pp. 1375-1380.

[4] Hraster, J.A., Memorandum to H.C. Hoffman, NASA Goddard Space Flight Center, Nov. 1972.